Basic AC Theory

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FREE ELECTRIC POWER?

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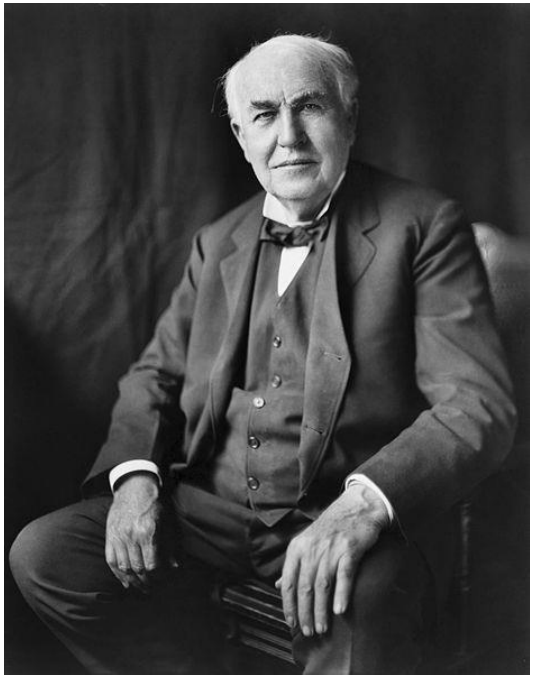
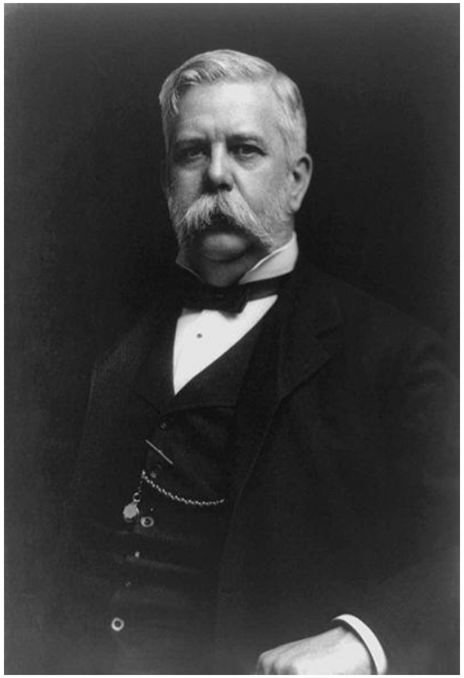
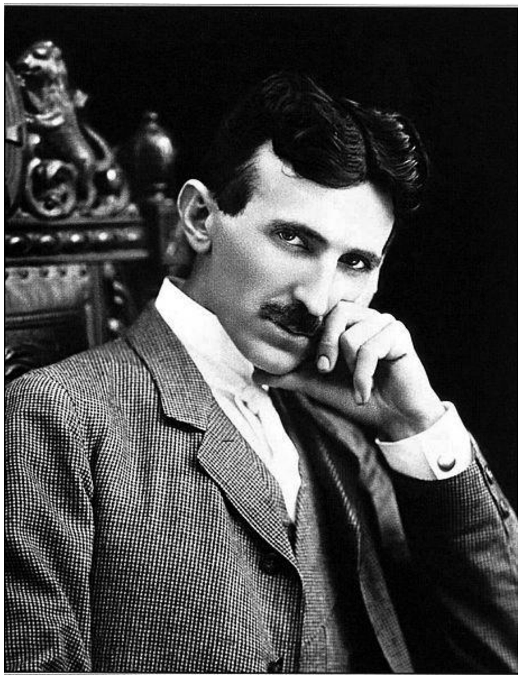
War of the Currents



Thomas Edison



George Westinghouse



Nikola Tesla

AC Theory - History

- Edison and Westinghouse
  - Edison favored DC power distribution, Westinghouse championed AC distribution.
  - The first commercial electric systems were Edison's DC systems.
- First AC system was in 1893 in Redlands, CA. Developed by Almirian Decker it used 10,000 volt, three phase primary distribution.
- Siemens, Gauland and Steinmetz were other pioneers.

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AC Theory - History

- AC eventually won out for power distribution.
  - Transformers allowed more efficient distribution of power over large areas.
  - AC motors were cheaper and easier to build.
  - AC electric generators were easier to build.

AC vs DC

- **Direct Current (DC)** – an electric current that flows in one direction.(IEEE100)

- **Alternating Current (AC)** – an electric current that reverses direction at regularly recurring intervals of time. (IEEE100)

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AC Circuits

- An AC circuit has three general characteristics
  - Magnitude
  - Frequency
  - Phase
- In the US, the household magnitude is 120 Volts with other common voltages being 208, 220, 277 and 480 Volts. The frequency is 60 Hertz (cycles per second).
- In a single phase system the relevant phase is current with respect to voltage.

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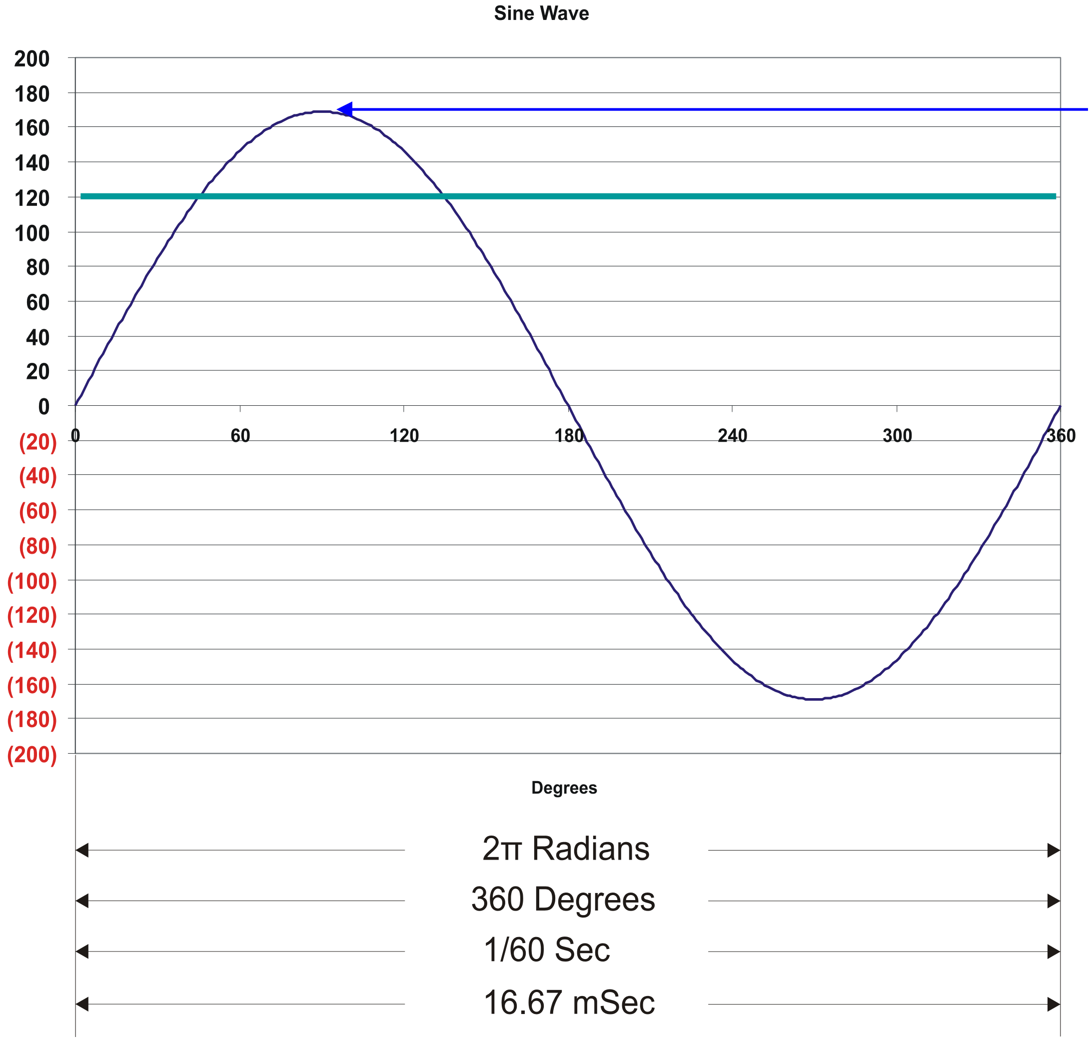
AC Theory – Sine Wave



$$V = V_{pk} Sin(2\pi ft - \theta)$$

$$V = \sqrt{2} V_{rms} Sin(2\pi ft - \theta)$$

$$V_{rms} = 120$$

$$V_{pk} = 169$$

$$\theta = 0$$

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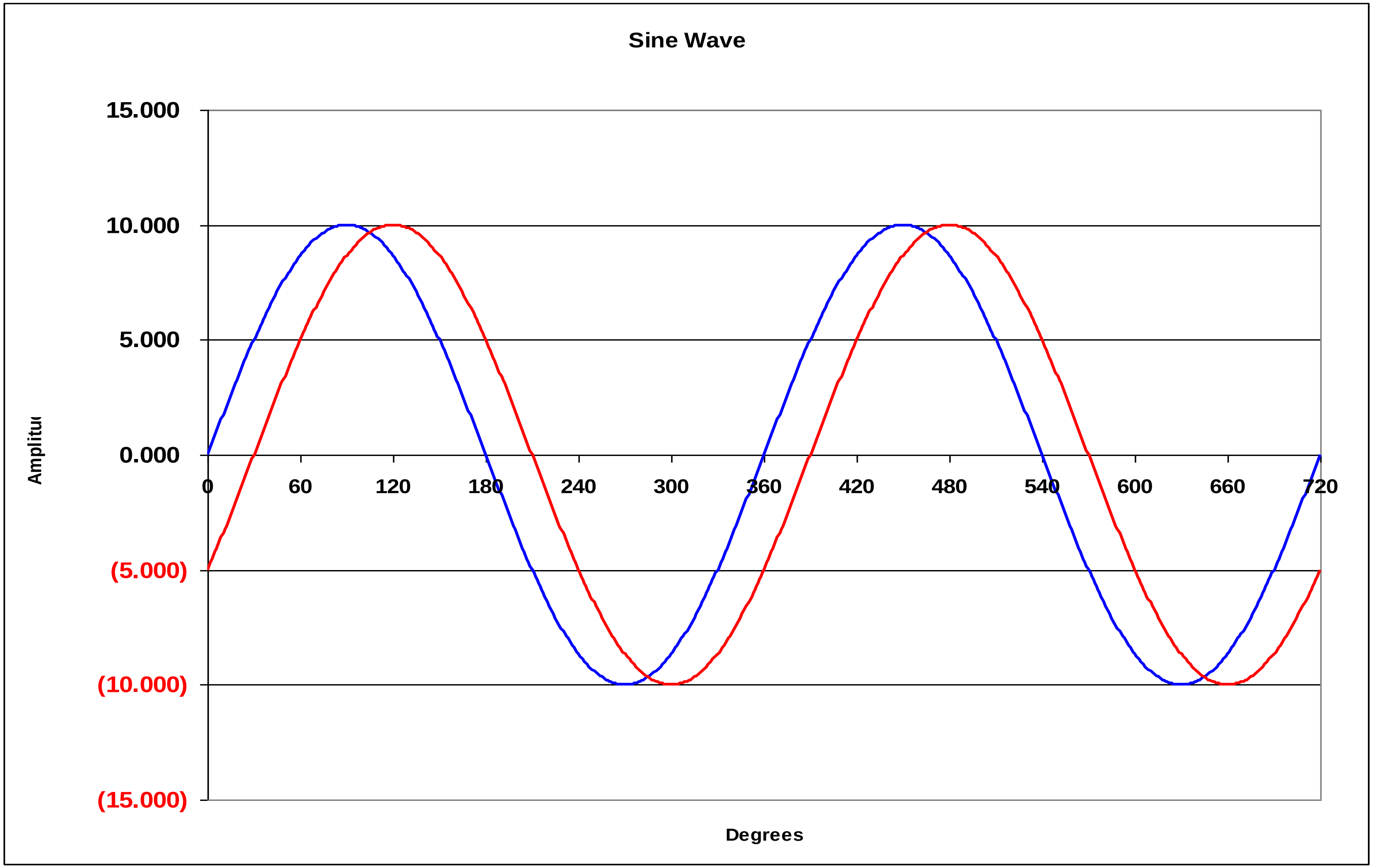
$V = 10Sin(\omega t - \theta)$

AC Theory - Phase



$\boxed{V = 10Sin(2\pi ft)}$ $\boxed{V = 10Sin(2\pi ft - 30)}$

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AC vs DC

- In DC theory we learned
  - Ohm's Law
    - **Voltage = Current x Resistance V = IR**
  - Power
    - **$P = I^2R = V^2/R$**

- For AC we would like the same equations to apply.
  - Specifically we want to be able to say that a DC voltage of 10 Volts applied to a resistor of value R produces the same power dissipation as an AC voltage of 10 volts applied to the same resistor.

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AC Theory – RMS

- For DC voltage to equal AC voltage we need

$$\frac{V_{dc}^2}{R} = \int \frac{1}{R} V_0^2 Sin^2(2\pi ft - \theta)dt$$

$$\frac{V_{dc}^2}{R} = \frac{V_0^2}{2R}$$

$$V_0 = \sqrt{2} V_{DC}$$

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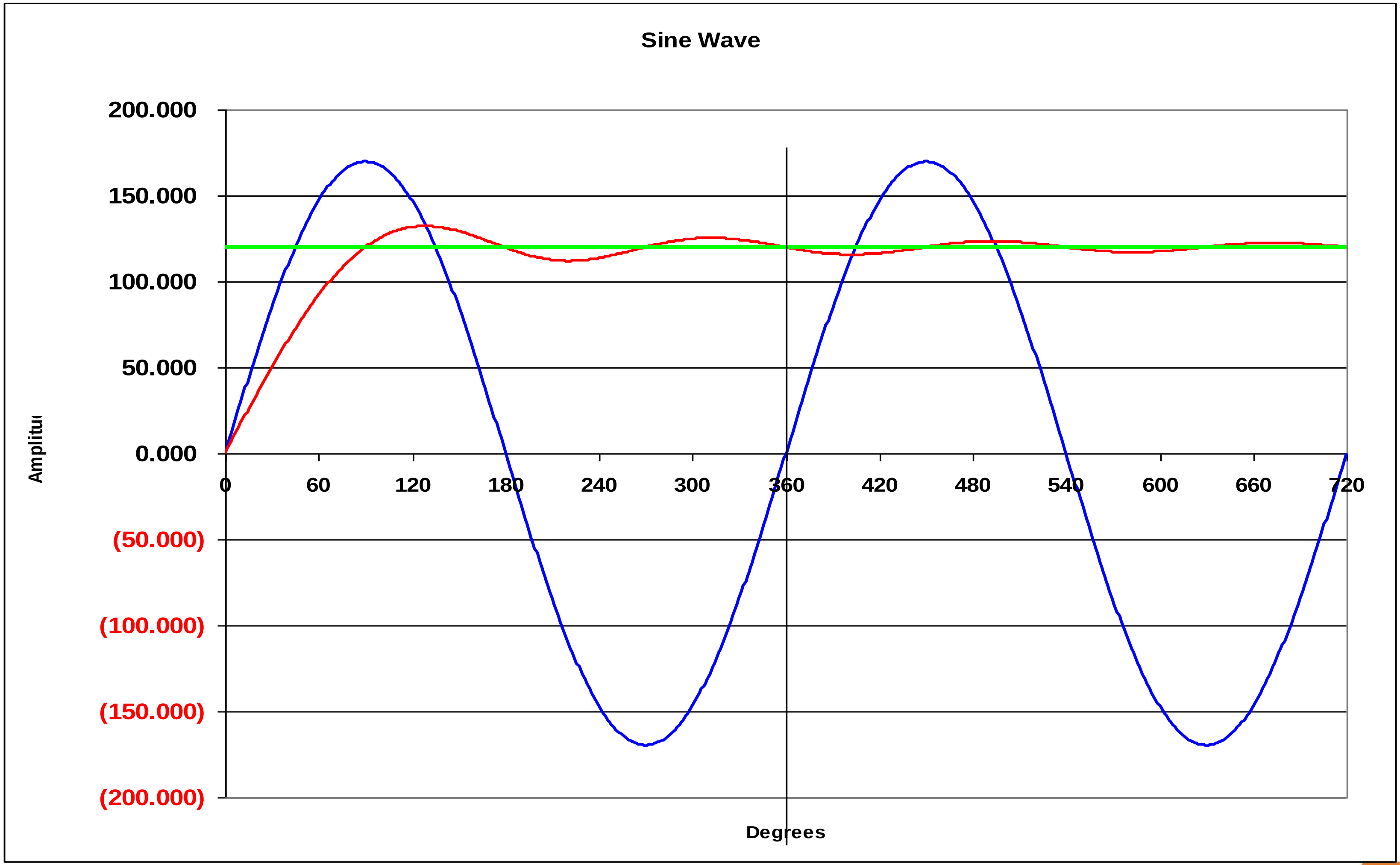
AC Theory - RMS



$$V = 120\sqrt{2}Sin(\omega t) = 169.68Sin(\omega t)$$

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AC Theory – RMS

- So if we want to have the V_0 in our equation for an AC signal represent the same value as the its DC counterpart we have

$$V = \sqrt{2} V_{DC} Sin(2\pi ft - \theta)$$

- By convention in AC theory we refer to V_{DC} as the RMS (Root Mean Squared) voltage.
- When we talk about AC values we always mean the RMS value not the peak value unless we say so specifically

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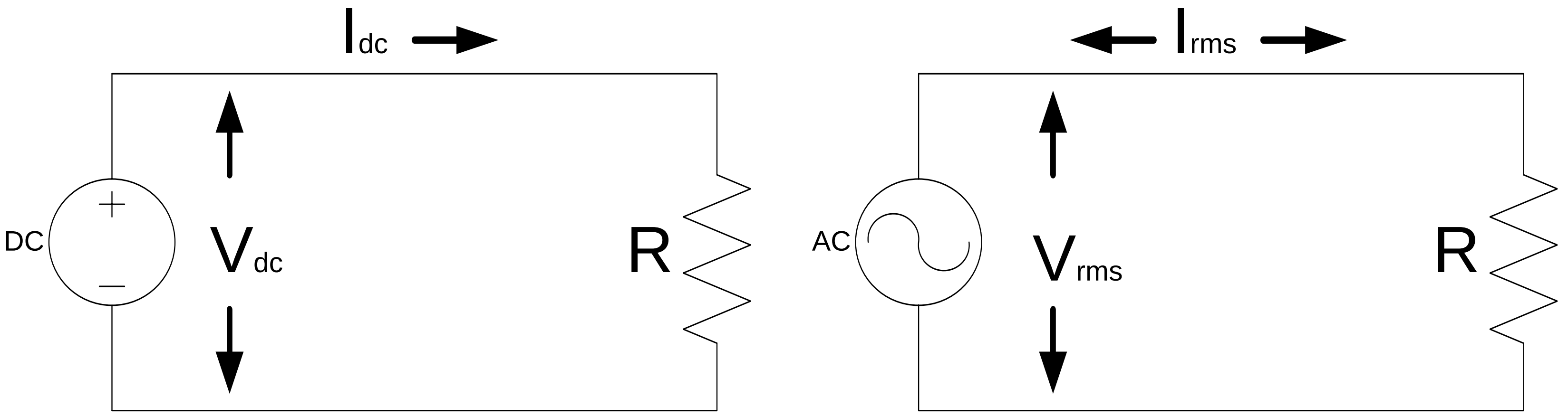
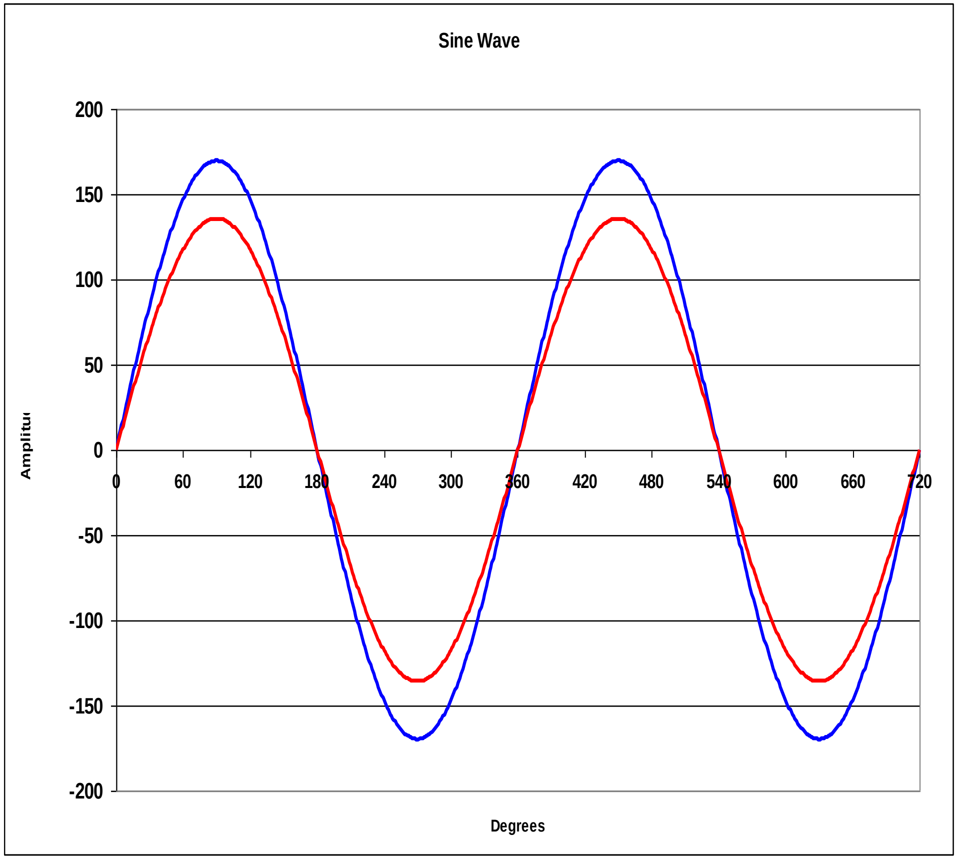
AC vs DC

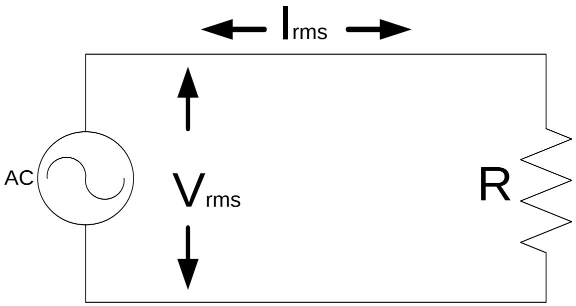


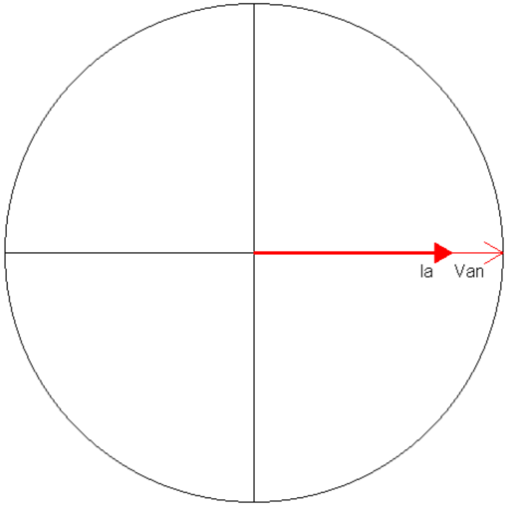
$$V = IR$$

$$P = VI = I^2R = V^2/R$$

AC Theory – Resistive Load

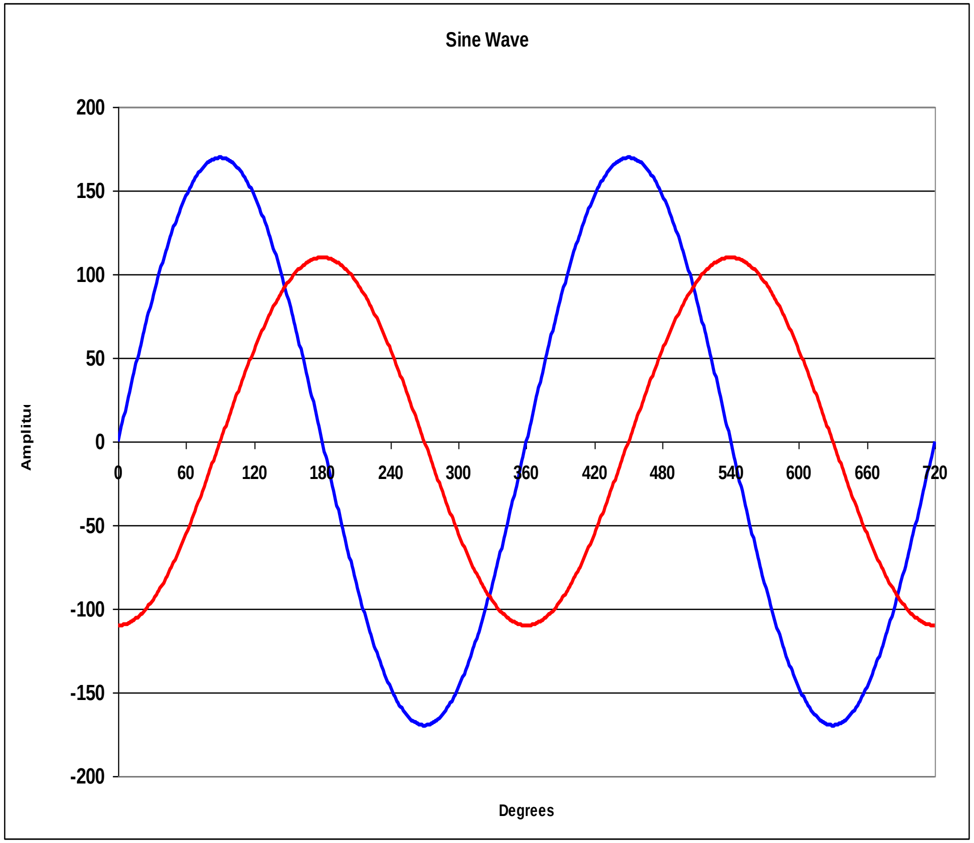


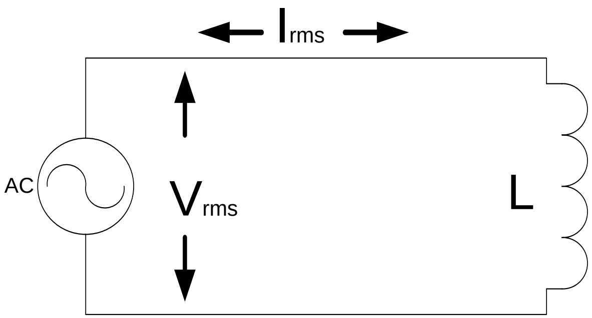


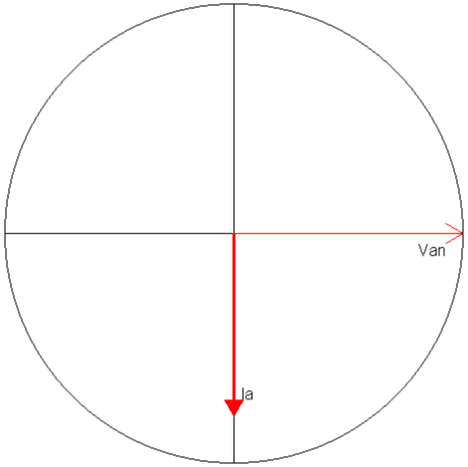


Resistors are measured in Ohms. When an AC voltage is applied to a resistor, the current is in degrees. A resistive load is considered a "linear" load because when the voltage is sinusoidal the current is sinusoidal.

AC Theory – Inductive Load

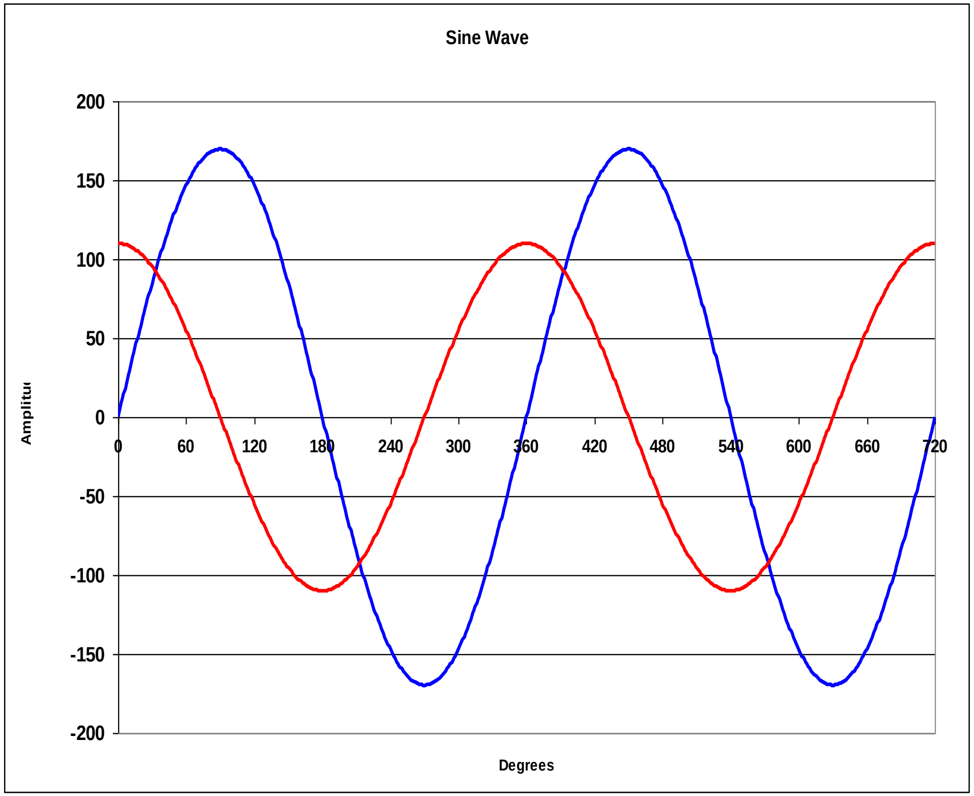


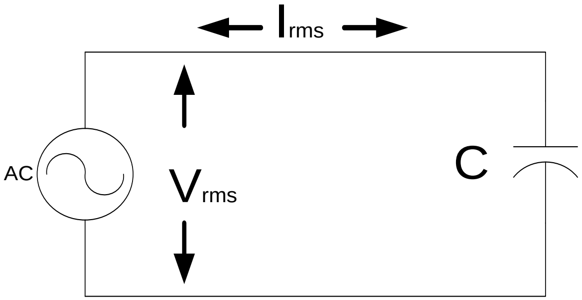


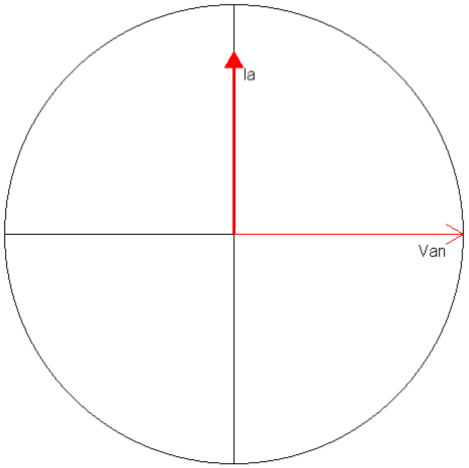


Inductors are measured in Henries. When an AC voltage is applied to an inductor, the current is 90 degrees out of phase. We say the current “lags” the voltage. A inductive load is considered a “linear” load because when the voltage is sinusoidal the current is sinusoidal.

AC Theory – Capacitive Load







Capacitors are measured in Farads. When an AC voltage is applied to a capacitor, the current is 90 degrees out of phase. We say the current “leads” the voltage. A capacitive load is considered a “linear” load because when the voltage is sinusoidal the current is sinusoidal.

AC Theory – Instantaneous Power

- **Active Power** is defined as P = VI
- Power is a rate, i.e. Energy per unit time.
- The Watt is the unit for Power
  - 1 Watt = 1000 Joules/sec
- Since the voltage and current at every point in time for an AC signal is different, we have to distinguish between **instantaneous power** and **average power**.
- Generally when we say “power” we mean average power.

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AC Theory – Energy

- Energy is power integrated over a period of time.
- The units of Energy are:
  - Watt-Hour (abbreviated Wh)
  - Kilowatt-Hour (abbreviated kWh)
- A Wh is the total energy consumed when a load draws one Watt for one hour.

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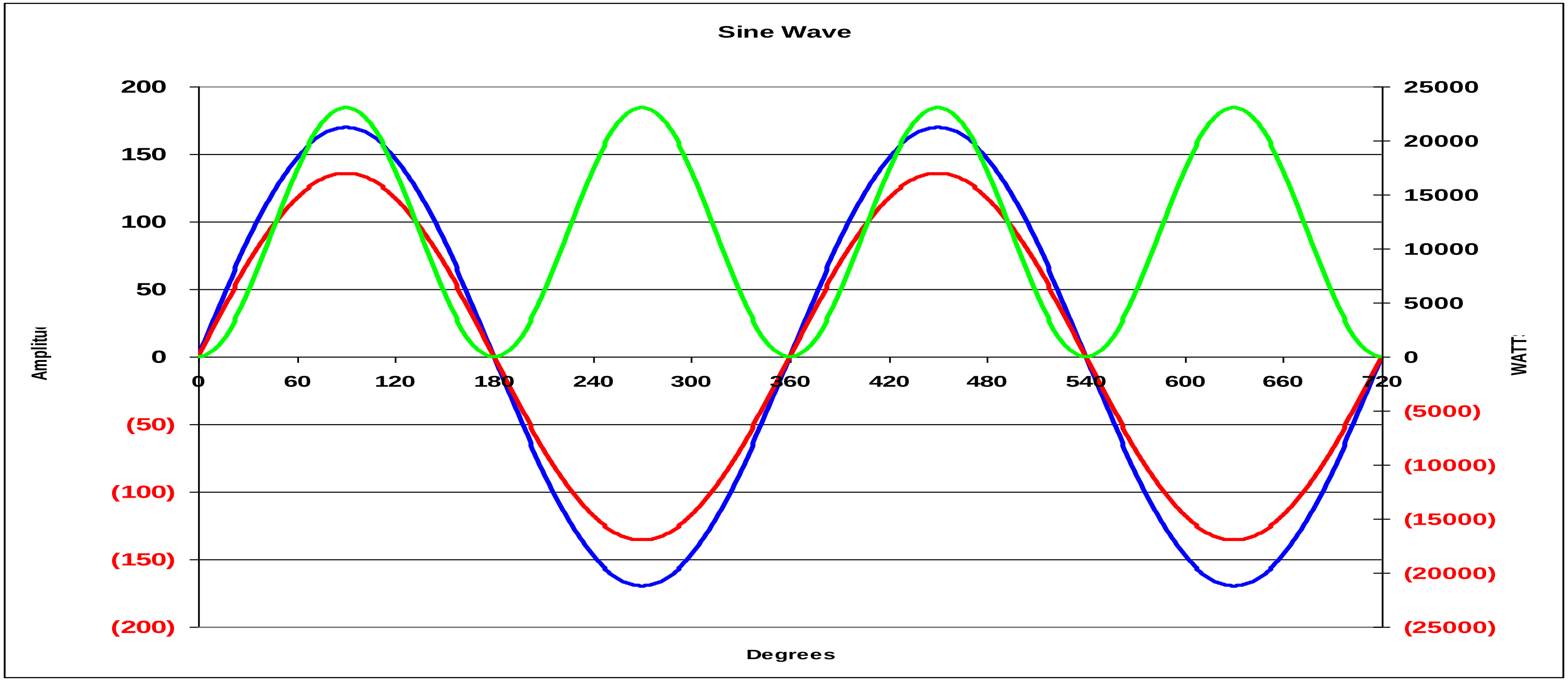
AC Theory – Instantaneous Power

For a resistive load: $p = vi = 2VISin^2(\omega t) = VI(1 - Cos(2\omega t))$



$V = 120\sqrt{2}Sin(2\pi ft)$

$I = 96\sqrt{2}Sin(2\pi ft)$

$P = 23040Sin^2(2\pi ft)$

P = 11520 Watts

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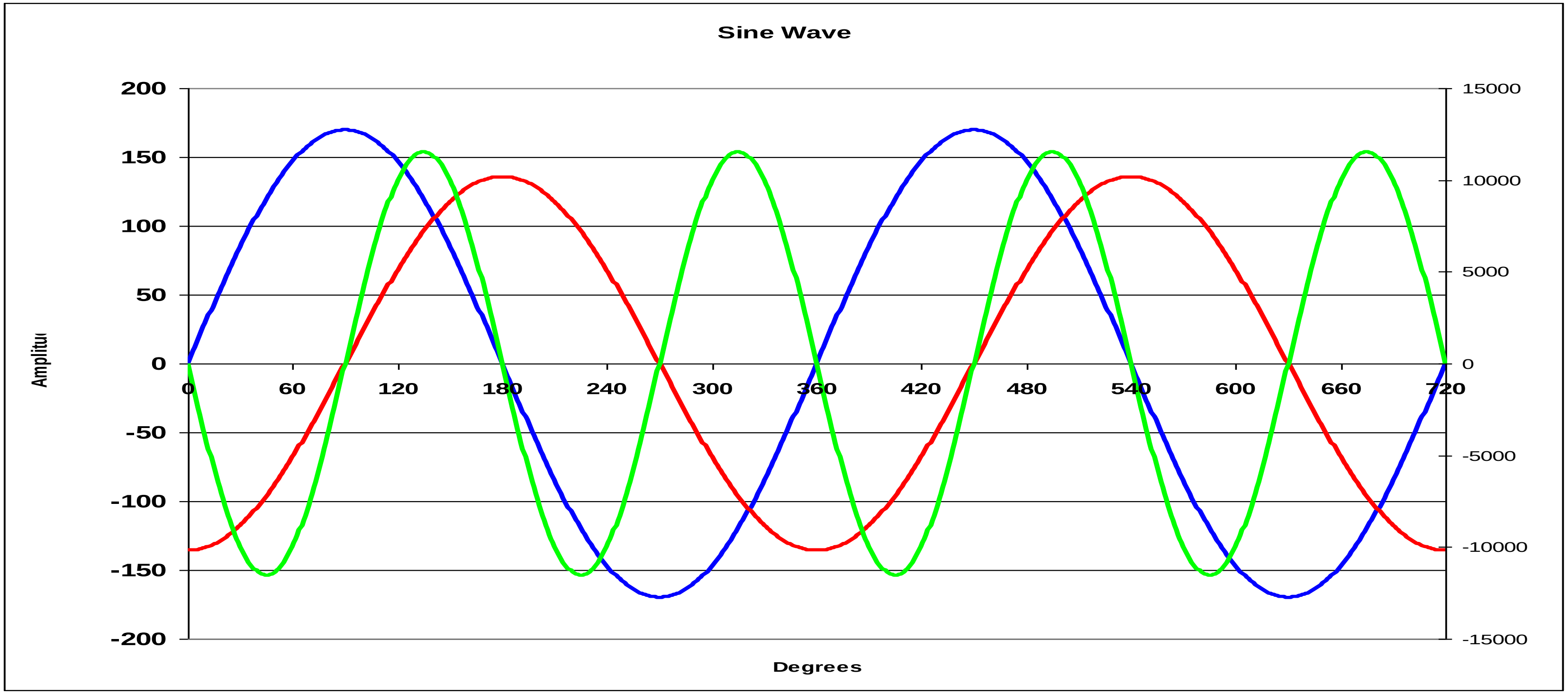
AC Theory – Instantaneous Power

For an inductive load: $p = vi = 2VISin(\omega t)Sin(\omega t - 90) = -VISin(2\omega t)$



$V = 120\sqrt{2}Sin(2\pi ft)$

$I = 96\sqrt{2}Sin(2\pi ft - 90)$

$P = -11520Sin(2\pi ft)$

P = 0 Watts

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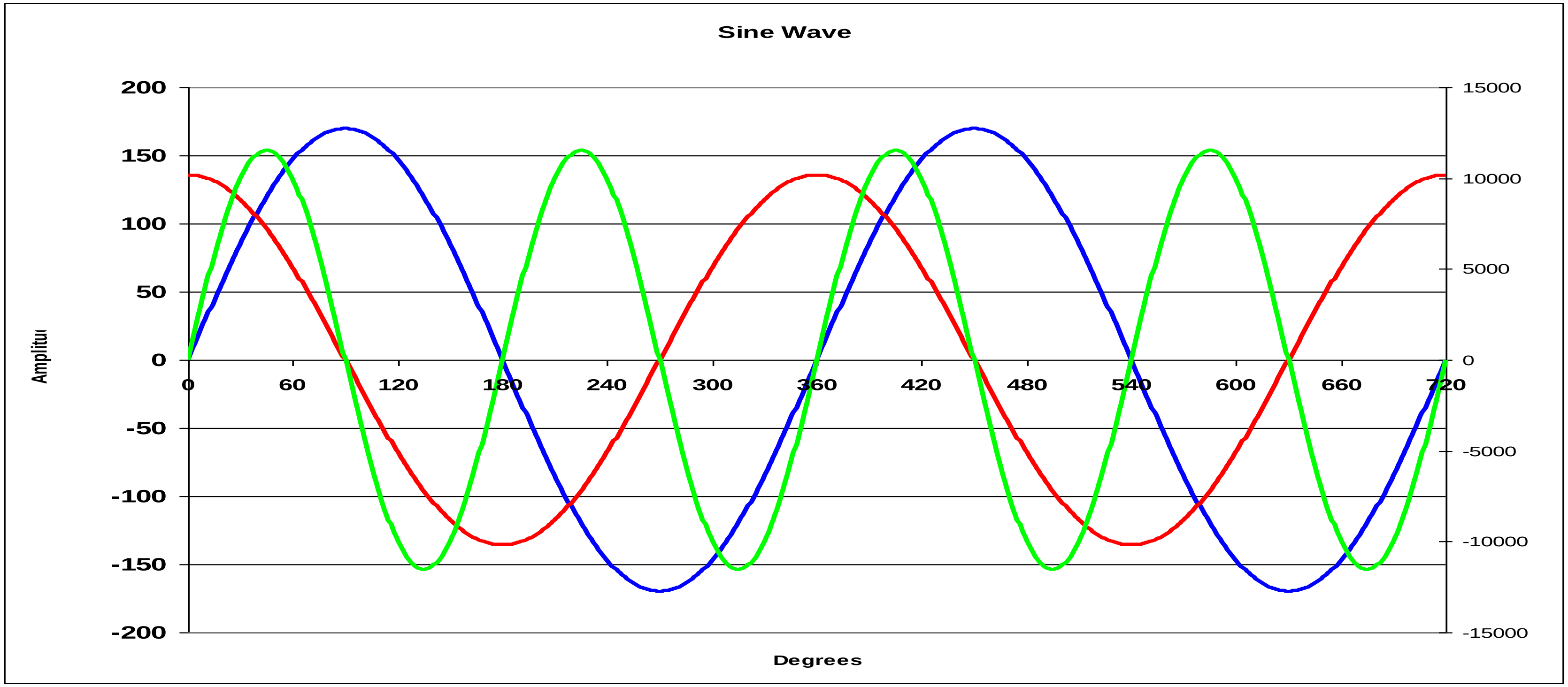
AC Theory – Instantaneous Power

For an capacitive load: $p = vi = 2VISin(\omega t)Sin(\omega t + 90) = VISin(2\omega t)$



$V = 120\sqrt{2}Sin(2\pi ft)$

$I = 96\sqrt{2}Sin(2\pi ft + 90)$

$P = 11520Sin(2\pi ft)$

P = 0 Watts

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AC Theory – Complex Circuits

- Impedance – The equivalent to the concept of resistance for an AC circuit. It is also measured in Ohms. Designated by the symbol X.
- In AC circuits non-resistive impedance affects both the amplitude and phase of the current.
- A resistor R has an impedance which is frequency independent. There is no phase shift.
- An inductor has an impedance which is proportional the frequency, $X_L = 2\pi fL$. The phase is shifted by 90 degrees lagging.
- A capacitor has an impedance which is inversely proportional the frequency, $X_C = 1/2\pi fC$. The phase is shifted by 90 degrees leading.

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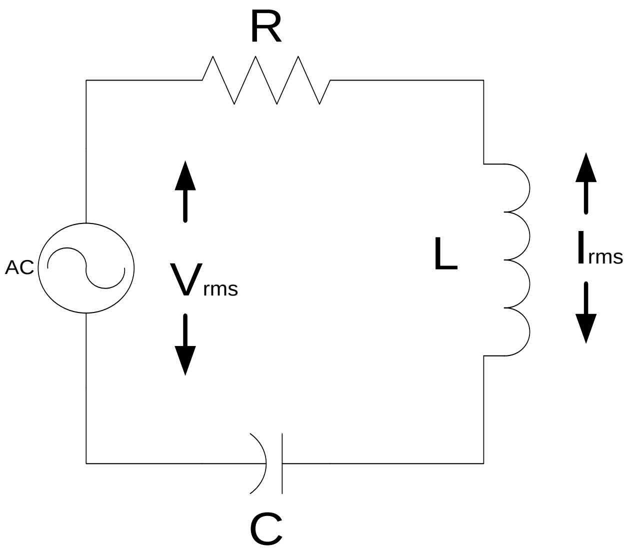
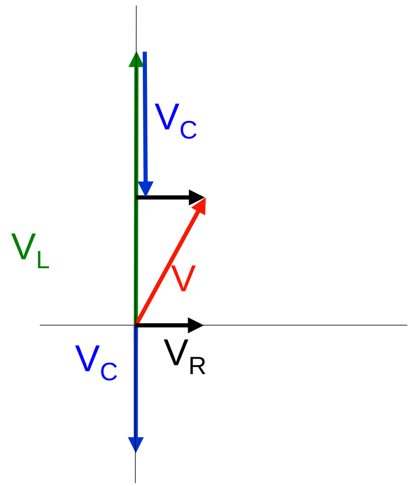
AC Theory – Complex Circuits





Amplitude (Current)

$$I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Phase (Current)

$$\varphi = ArcTan\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right]$$

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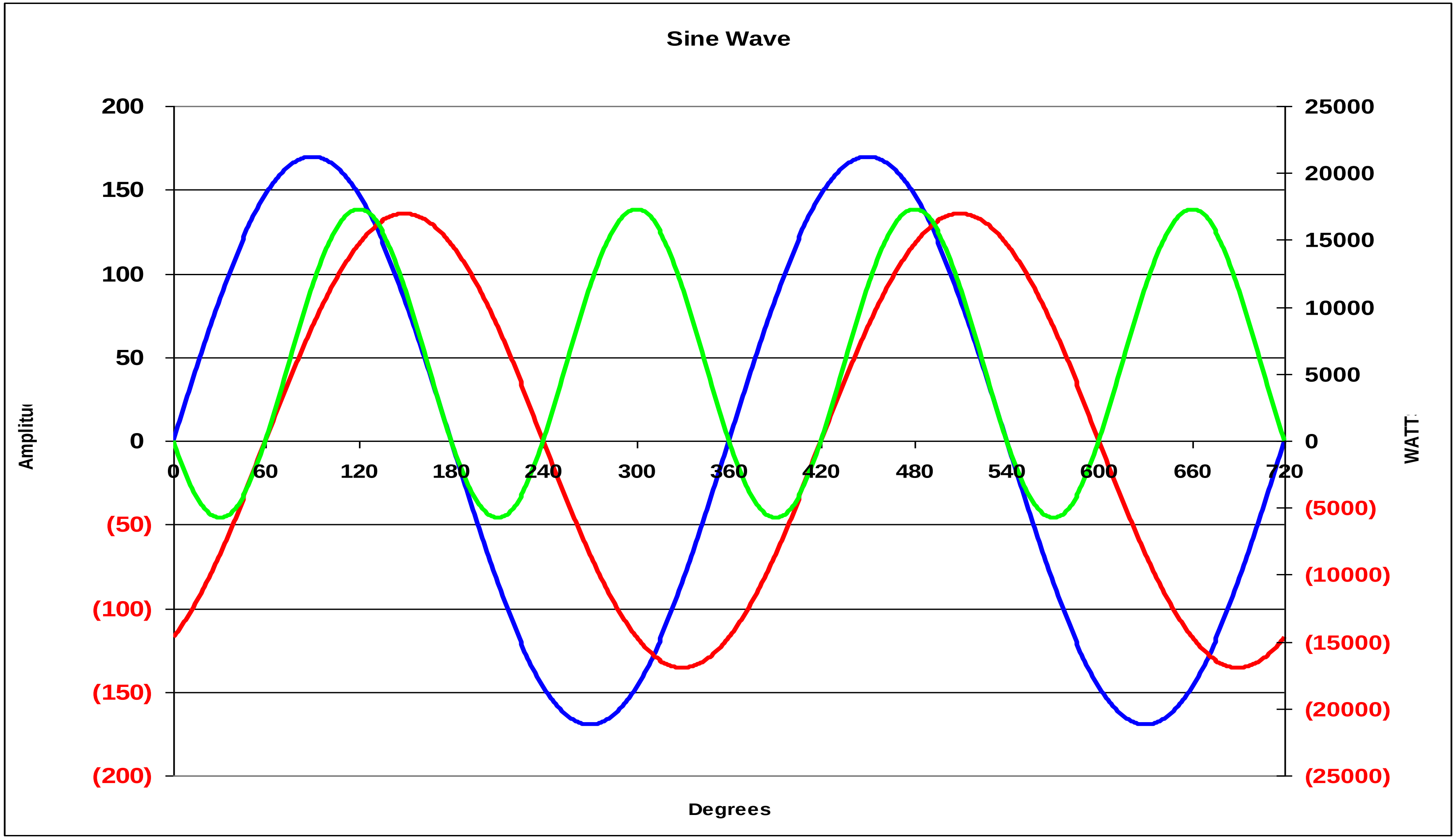
AC Theory – Instantaneous Power



$$V = 120\sqrt{2}Sin(2\pi ft)$$

$$I = 96\sqrt{2}Sin(2\pi ft - 60)$$

$$P = VI = 23040(Cos(60°) + Cos(4\pi ft - 60°)) = 19953 - 23040Cos(4\pi ft - 60°)$$

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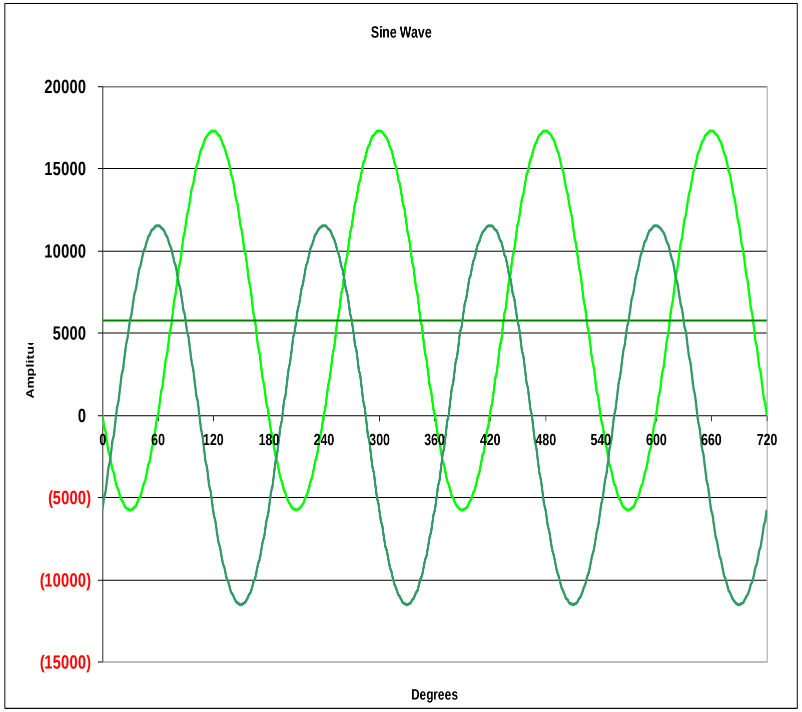
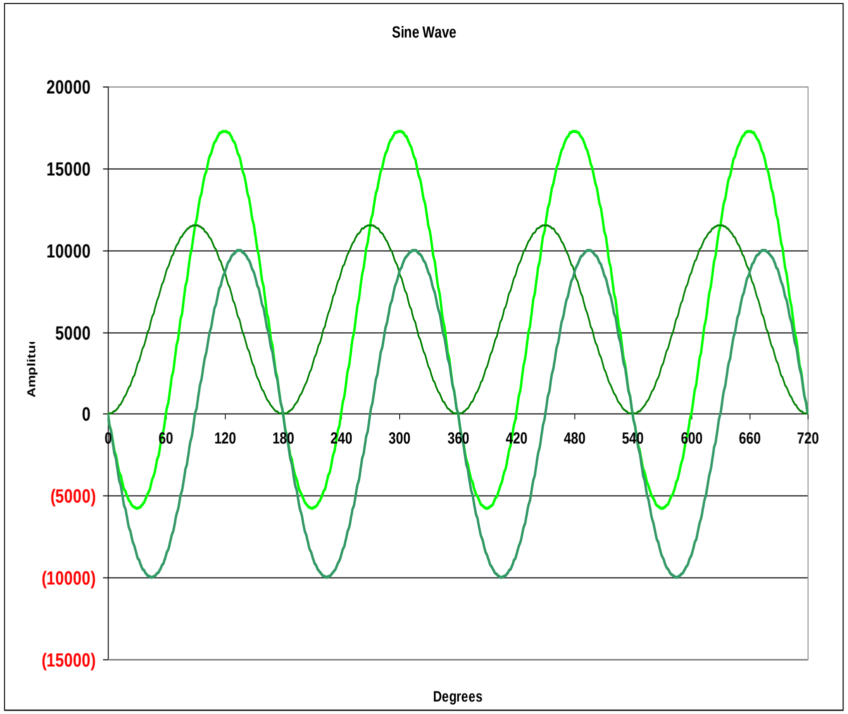
AC Theory – Instantaneous Power

- From IEEE1459 instantaneous power can be written in several forms:





Active Power Reactive Power

$$p = VI\cos\theta - VI\cos(2\omega t - \theta) \qquad p = VI\cos\theta[1 - \cos(2\omega t)] - VI\sin\theta\sin(2\omega t)$$

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Time Out for Trig

(Right Triangles)

The Right Triangle:

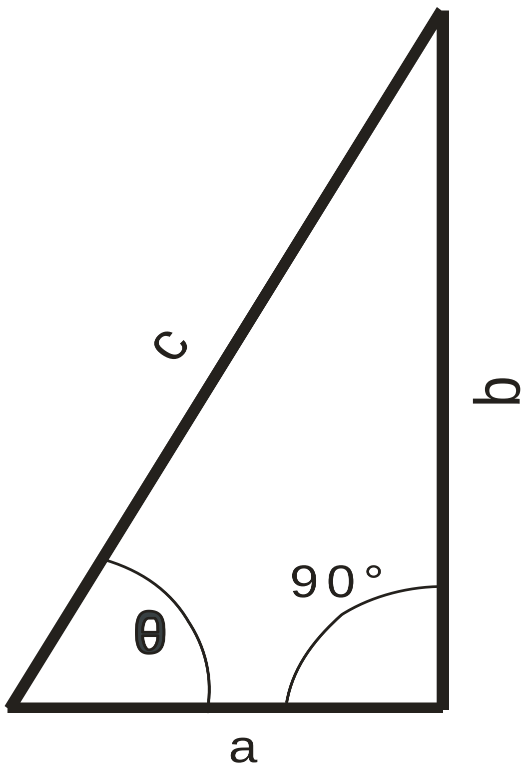
The Pythagorean theory

$$c^2 = a^2 + b^2$$

$$Sin(\theta) = \frac{b}{c}$$

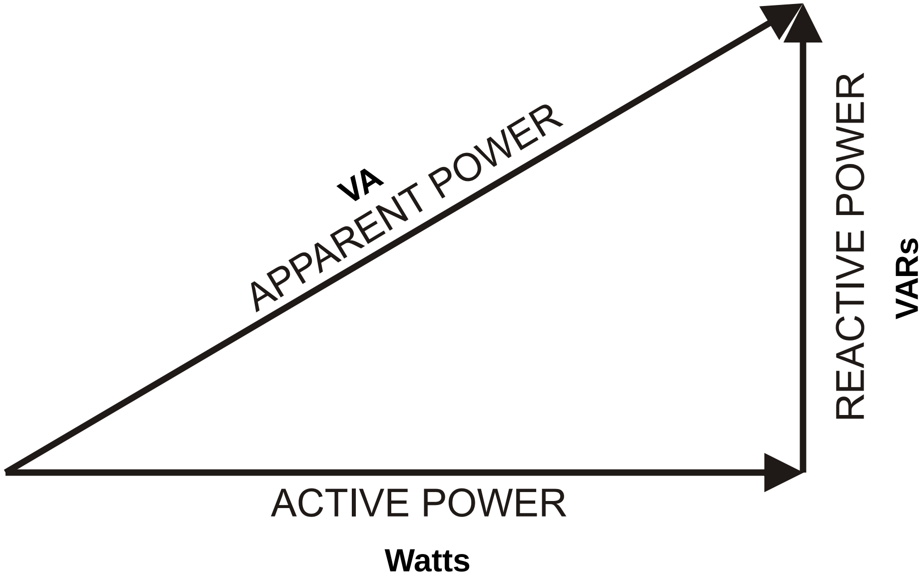
$$Cos(\theta) = \frac{a}{c}$$

$$Tan(\theta) = \frac{b}{a}$$



AC Theory – Power Triangle

(Sinusoidal Waveforms)



If V = Sin(ωt) and I = Sin(ωt - θ) (the load is linear) then

Active Power =	$VI\,Cos(\theta)$	Watts
Reactive Power =	$VI\,Sin(\theta)$	VARs
Apparent Power =	VI	VA
Power Factor =	Active/Apparent = $Cos(\theta)$	

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Harmonics

Curse of the Modern World

- Every thing discussed so far was based on “Linear” loads.
  - For linear loads the current is always a simple sine wave. Everything we have discussed is true.
- For nearly a century after AC power was in use ALL loads were linear.
- Today, many loads are NON-LINEAR.

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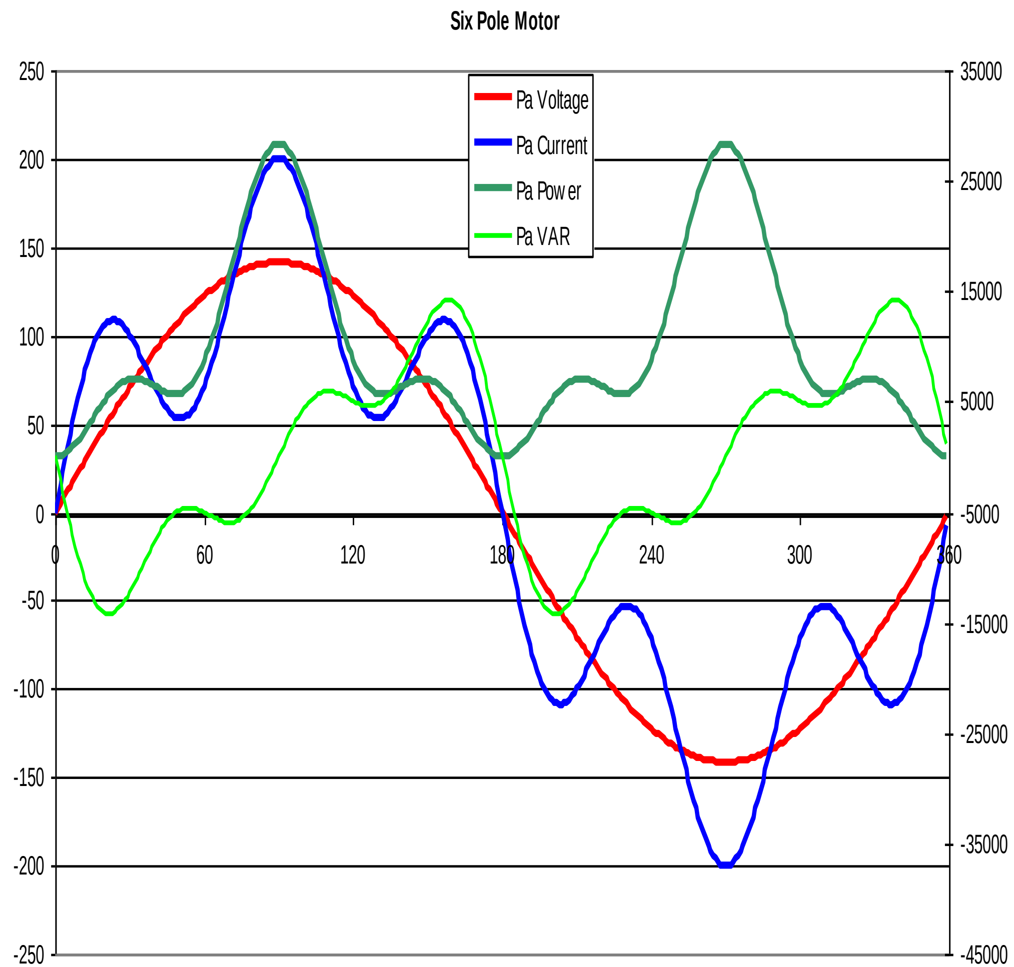
Harmonic Load Waveform

Eq.#	Quantity	Phase A
1	V(rms) (Direct Sum)	100
2	I(rms) (Direct Sum)	108
3	V(rms) (Fourier)	100
4	I(rms) (Fourier)	108
5	Pa = (∫ V(t)I(t)dt)	10000
6	Pb = ½∑V*n*In**cos**(θ)	10000
7	Q = ½∑VnIn**sin**(θ)	0.000
8	Sa = Sqrt(P^2 +Q^2)	10000
9	Sb = Vrms*Irms(DS)	10833
10	Sc = Vrms*Irms(F)	10833
13	PF = Pa/Sa	1.000
14	PF = Pb/Sb	0.923
15	PF = Pb/Sc	0.923



V = 100Sin(ωt)   I = 100Sin(ωt) + 42Sin(5 ωt)

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Harmonic Load Waveform

Eq.#	Quantity	Phase A
1	V(rms) (Direct Sum)	100
2	I(rms) (Direct Sum)	108
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6	Pb = ½∑V*n*In**cos**(θ)	10000
7	Q = ½∑VnIn**sin**(θ)	0.000
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10	Sc = Vrms*Irms(F)	10833
13	PF = Pa/Sa	1.000
14	PF = Pb/Sb	0.923
15	PF = Pb/Sc	0.923

- Important things to note:
  - Because the voltage is NOT distorted, the harmonic in the current does not contribute to active power.
  - It does contribute to the Apparent power.
  - The Power Triangle does not hold

$$S \neq \sqrt{P^2 + Q^2}$$

  - There is considerable disagreement about the definition of various power quantities when harmonics are present.

$V = 100\text{Sin}(\omega t)$ $I = 100\text{Sin}(\omega t) + 42\text{Sin}(5\ \omega t)$

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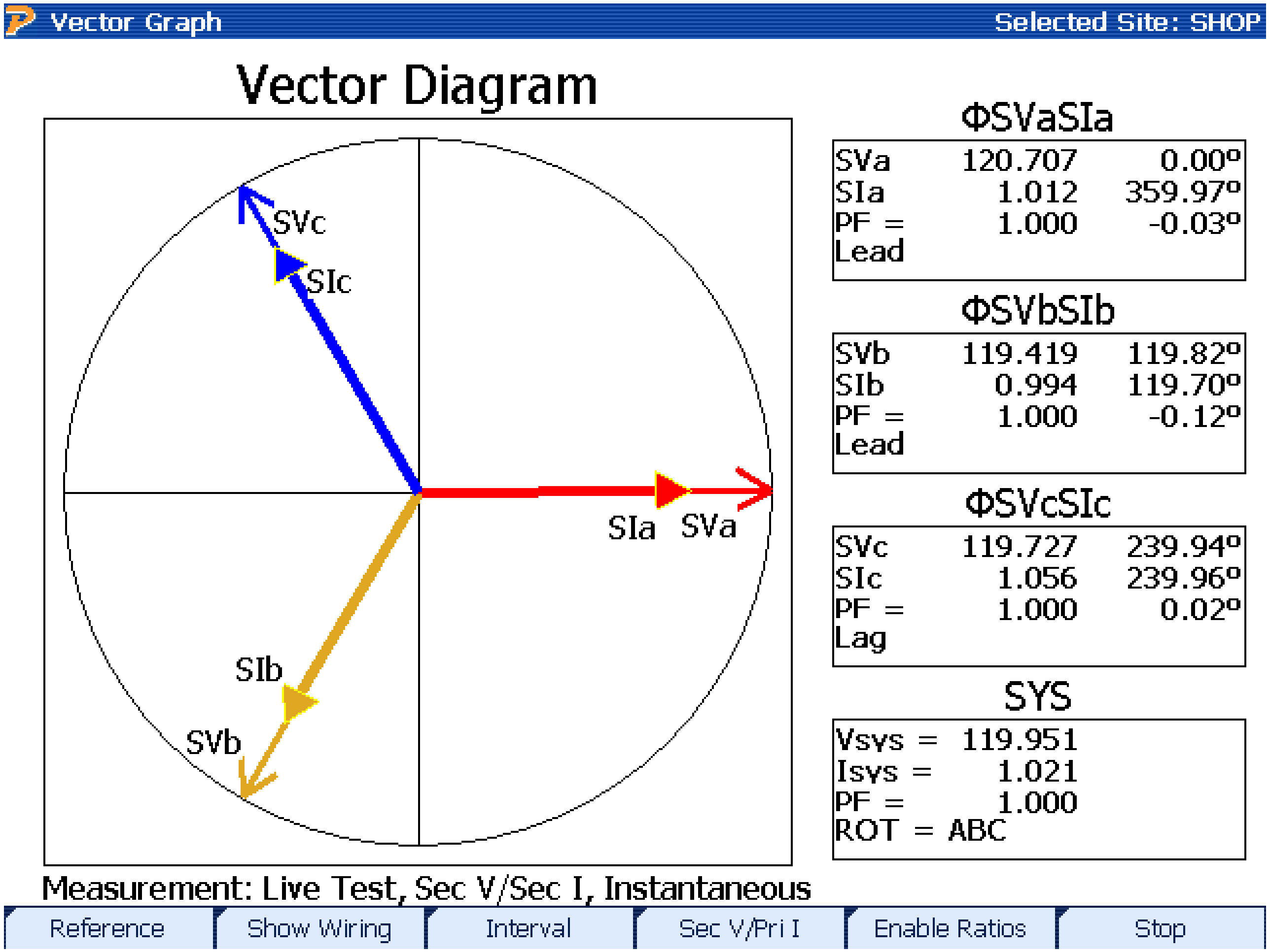
AC Theory - Phasors

- An easier way to view AC data



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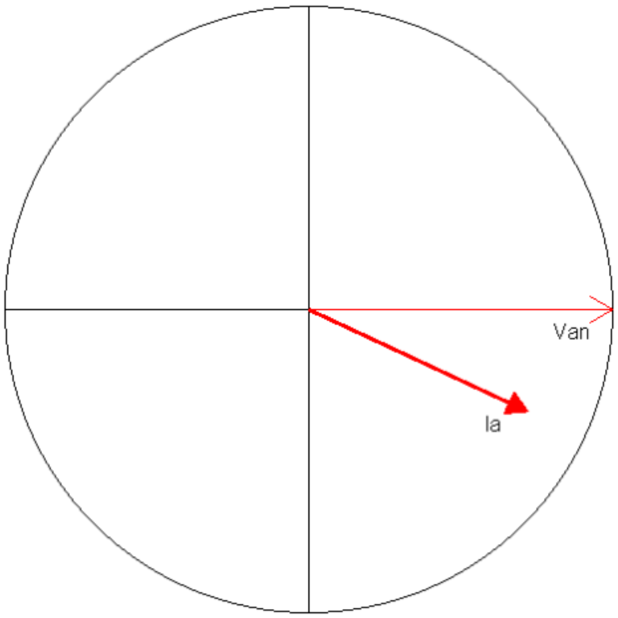
AC Theory - Phasors

- The length of the phasor is proportional to the value of the quantity
- The angle of the phasor (by convention phase A is drawn as horizontal) shows the phase of the quantity relative to phase A voltage.
- Here the current "lags" the voltage by 25 degrees.



$$V = 120\sqrt{2}Sin(2\pi ft - 0)$$

$$I = 2.5\sqrt{2}Sin(2\pi ft - 25)$$

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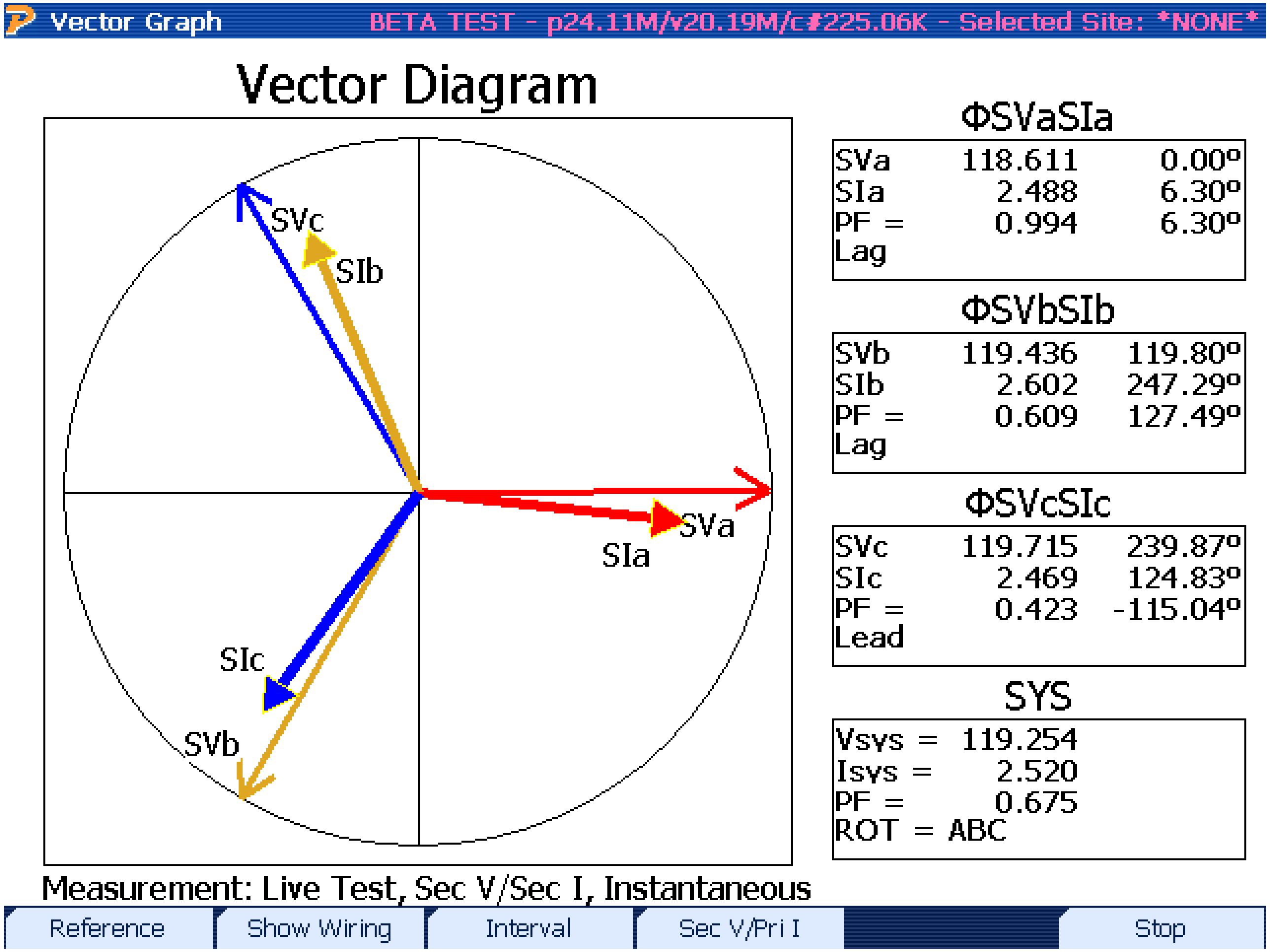
AC Theory - Phasors

- Phasors are particularly useful in poly-phase situations



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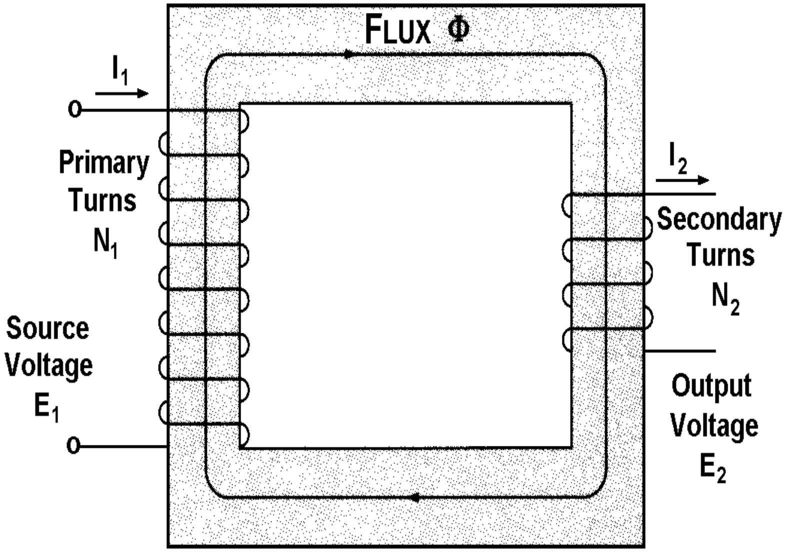
What is a Transformer?

- A TRANSFORMER is a device used to change the voltage levels of electricity to facilitate the transfer of electricity from generating stations to customers. A step-up transformer increases the voltage while a step-down transformer decreases it. www.duquesnelight.com/understandingelectricityupdate/electricterms.html



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Basic Transformer Theory

- Vp = primary voltage
- Ip = primary current
- Np = primary turns
- Pp = primary power
- Vs = secondary voltage
- Is = secondary current
- Ns = secondary turns
- Ps = secondary power

$$Vs = \frac{Ns}{Np} Vp$$

$$Is = \frac{Np}{Ns} Ip$$

$$Pp = Vp \bullet Ip = Ps = Vs \bullet Is$$

This is true for an IDEAL transformer!

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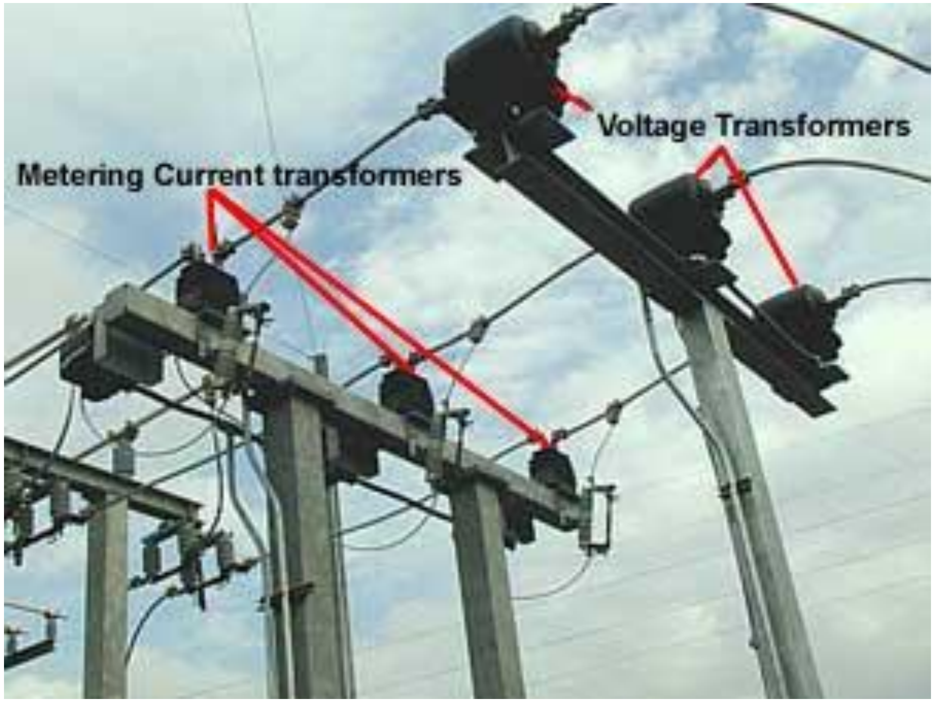
What is an Instrument Transformer?

Instrument Transformers convert signal levels from dangerous (high voltage) or inconvenient (high current, or current at high voltage) to levels appropriate for metering.



There are two fundamental types:

CT’s (Current Transformers)

PT’s (Potential Transformers)

Potential Transformers (PT's)

- PT's step down high voltages to the voltage needed by the meter (usually 120V occasionally 67V).
- They come in many shapes and sizes for different applications
- They work exactly as you would expect them to: Vo=Vi•(Ns/Np).
- They come in various power ratings expressed in VA.
- They come in various accuracy classes, however the 0.3% accuracy class is generally used in North America.

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Current Transformers (CT's)

- CT's allow the measurement of high currents at potentially high voltages.
- They come in many shapes and sizes for different applications
- They are potentially extremely dangerous.

They can kill you!

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Errors with Instrument Transformers

CT - Polarity

- Polarity of the connection matters.
- Wrong polarity means totally wrong metering.
- When PF≠0, reversed polarities may not be obvious.

CT Shunt is VERY IMPORTANT!

- CT Secondary MUST be shunted before removing the meter or an ARC FLASH may occur!!!

SHUNT CLOSED

SHUNT OPEN

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